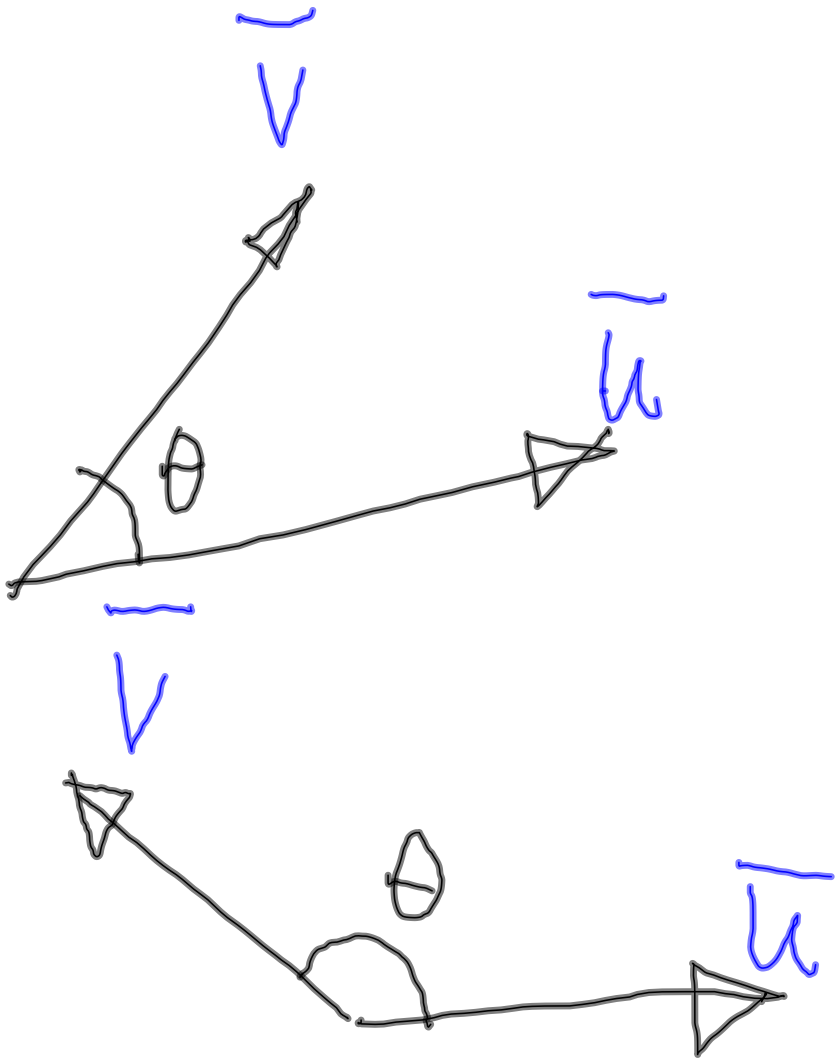
Skalärprodukt



nov 4-09:57

<u>längden</u>

$$|\bar{u}|^2 = \bar{u} \cdot \bar{u}$$

$$|(1,3,2)|^2 =$$

$$(1,3,2) \cdot (1,3,2)$$

$$= 1 \cdot 1 + 3 \cdot 3 + 2 \cdot 2 = 14$$

$$|(1,3,2)| = \sqrt{14}$$

nov 4-09:58

Vi kan lösa ut

$\cos\theta$ som

$$\cos\theta = \frac{\bar{u} \circ \bar{v}}{|\bar{u}||\bar{v}|}$$

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Projektion

$$\text{Proj}_{\bar{v}}\,\bar{u} = \frac{\bar{u}\cdot\bar{v}}{\bar{v}\cdot\bar{v}}\,\bar{v}$$

Skalärer

längden för $\bar{v}$ kommer med två ggr i täljaren

nov 4-09:58

och två ggr i nämnaren.

Kolla hur det fungerar vid skalning

$\bar{u} \longmapsto a\bar{u}$

eller

$\bar{v} \longmapsto a\bar{v}$

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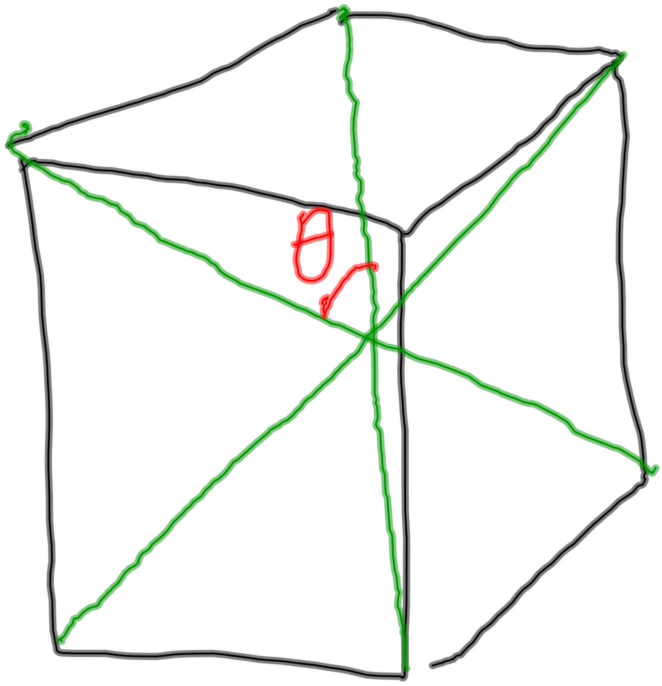

Vad är vinkeln ?
mellan diagonalerna

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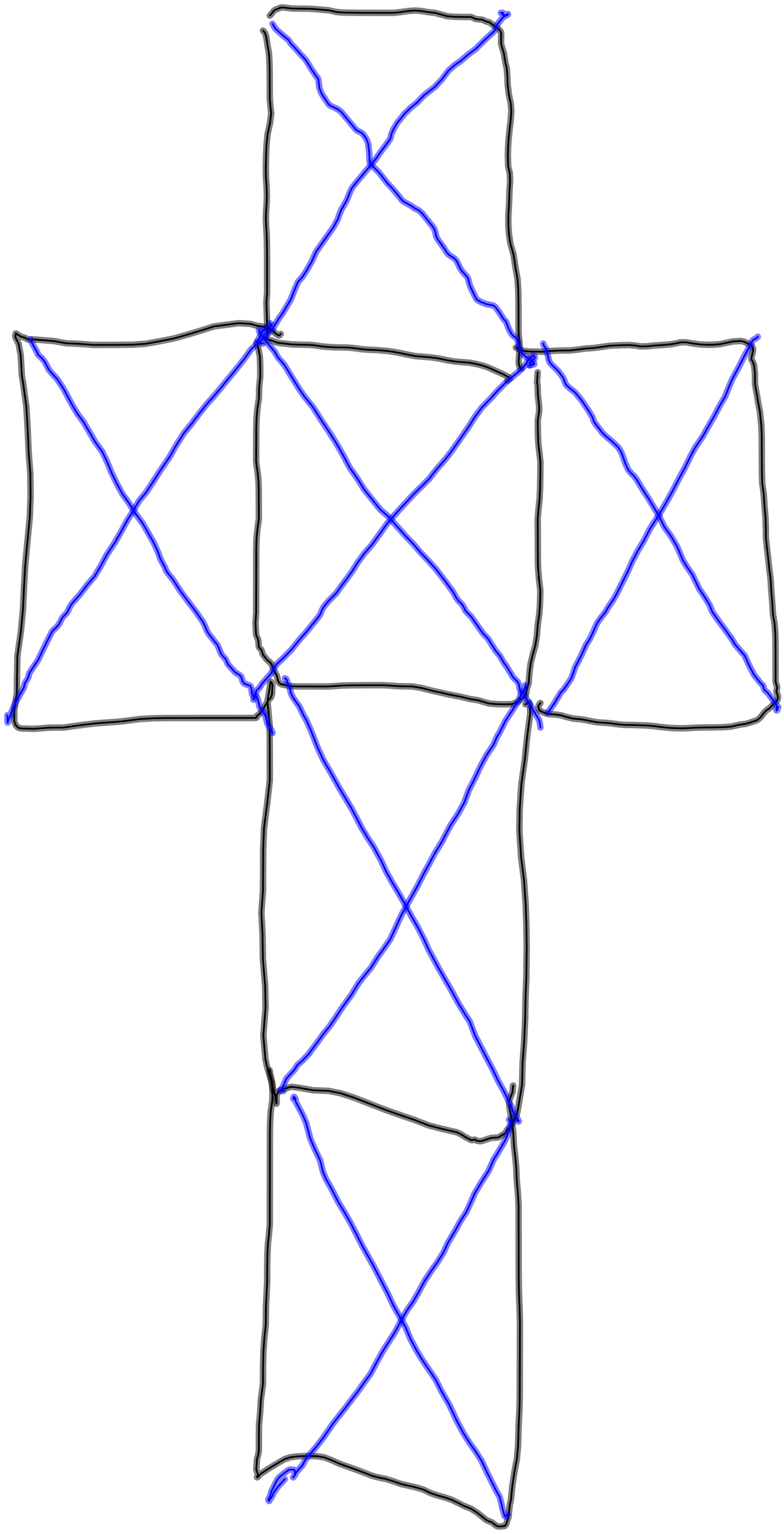

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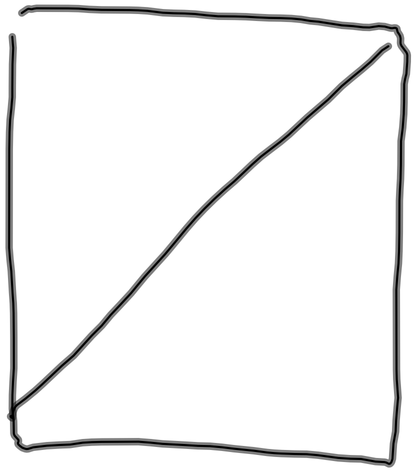
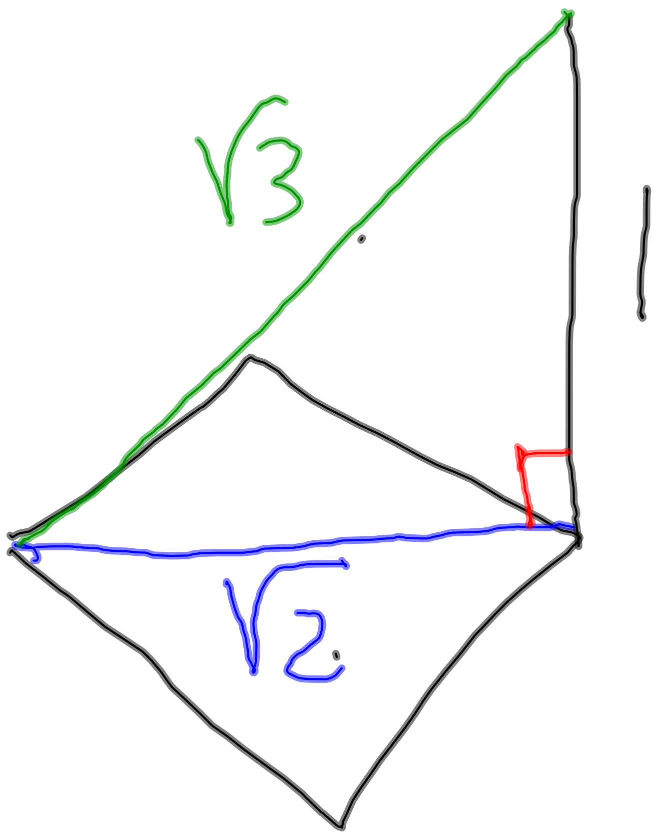
$$(\sqrt{3})^2 = 1^2 + (\sqrt{2})^2$$





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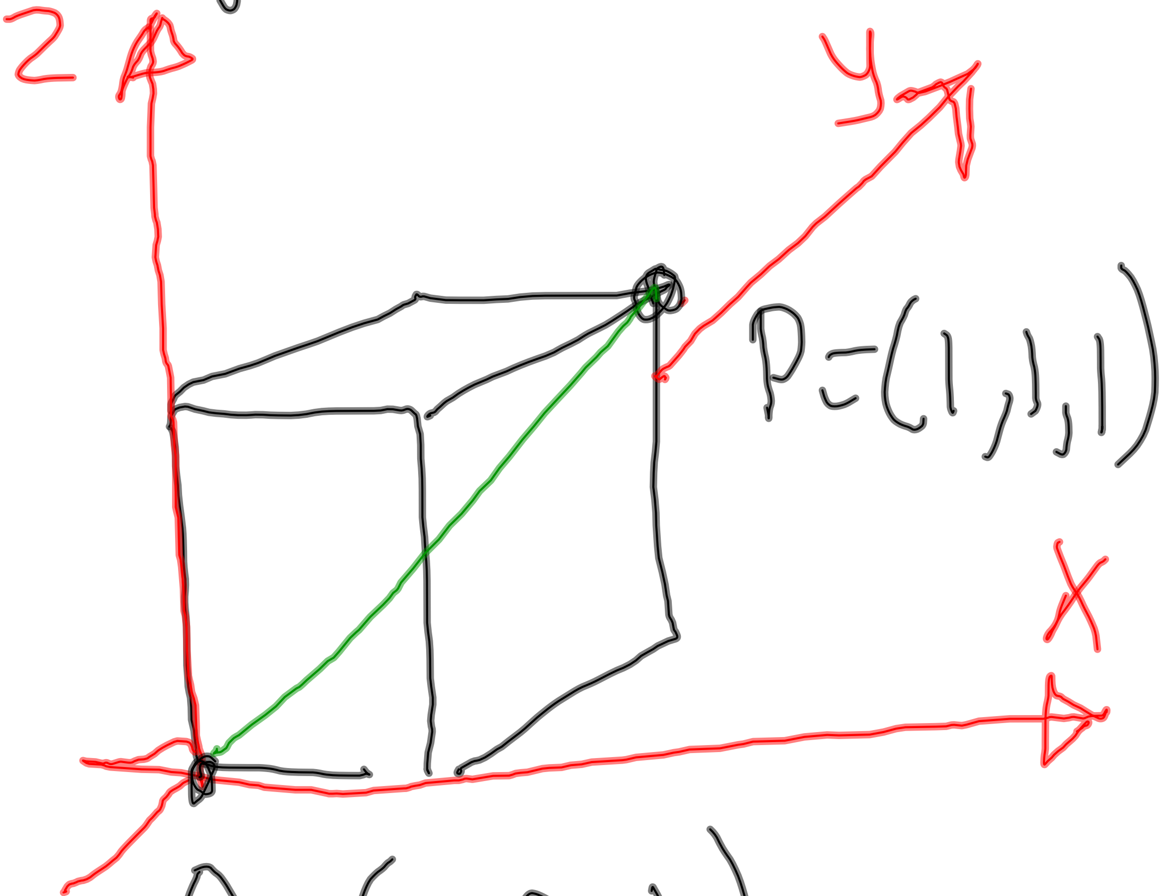
Inför koord.



$O=(0,0,0)$

$|\overrightarrow{OP}| = |(1,1,1)| =$

nov 4-10:25

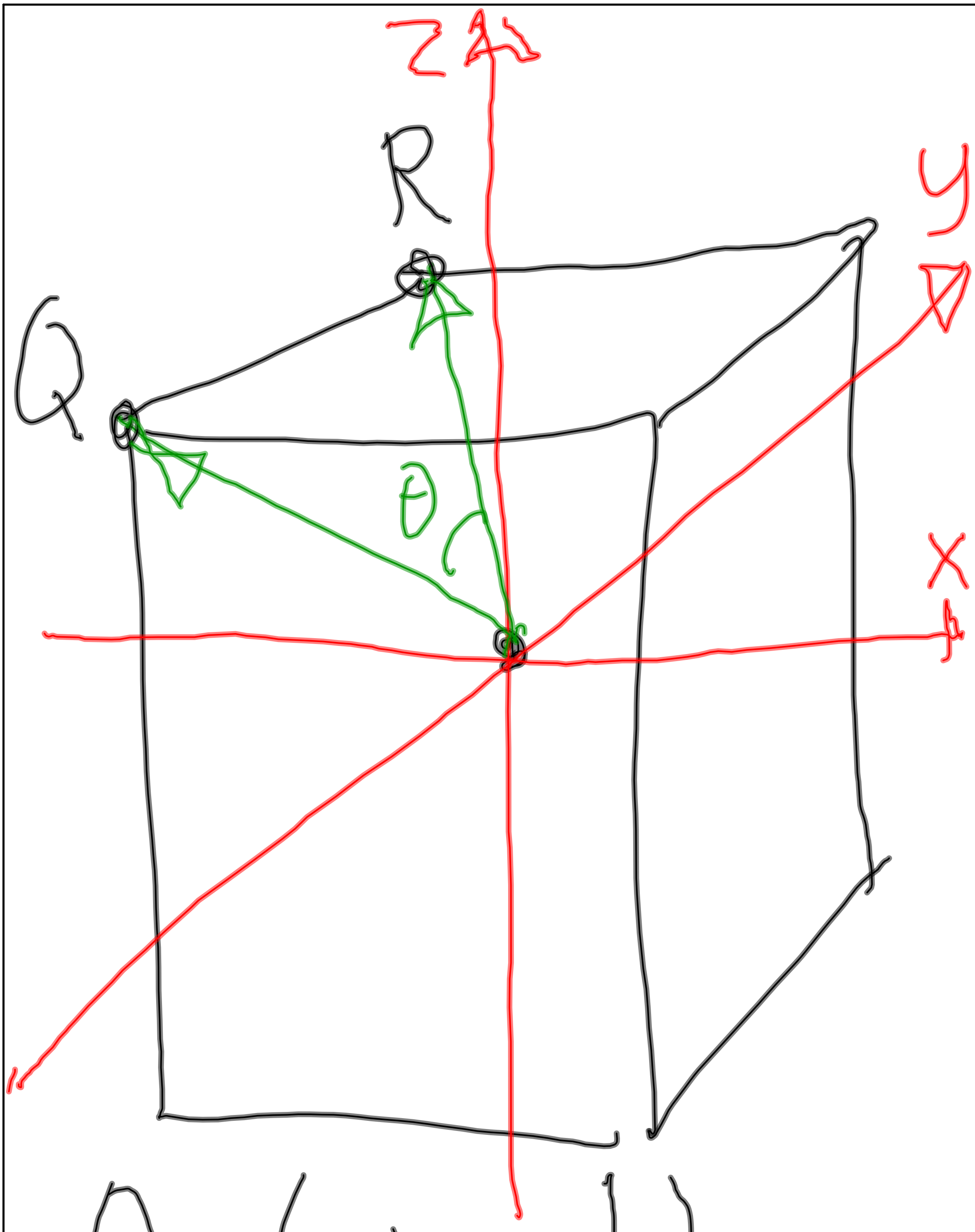
$$\sqrt{(1,1,1)\cdot(1,1,1)} =$$

$$\sqrt{1^2+1^2+1^2} = \sqrt{3}$$

nov 4-10:27

Vi kan sätta origo i mitten av en kub med sida 2

nov 4-10:28



$Q = (-1, 1, 1)$

$R = (-1, -1, 1)$

nov 4-10:29

$$\overrightarrow{OR} \cdot \overrightarrow{OQ} =$$

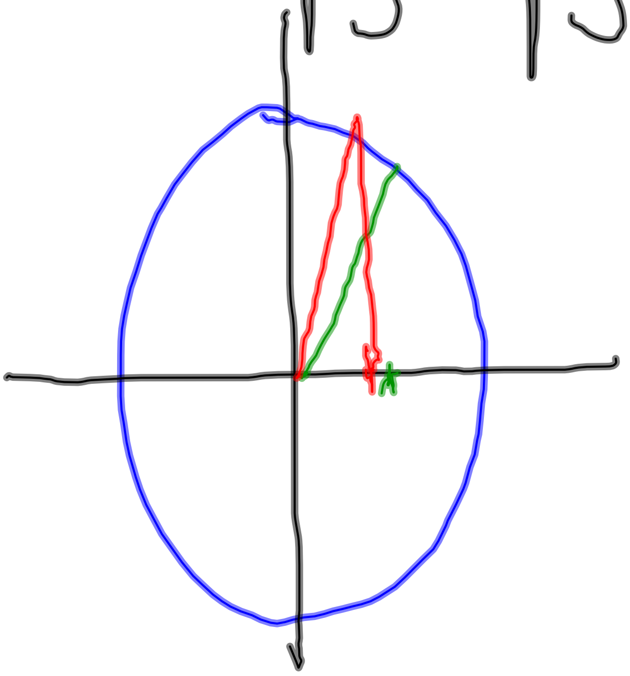
$$(-1, 1, 1) \cdot (-1, -1, 1)$$

$$= (-1)^2 + 1 \cdot (-1) + 1^2$$

$$= 1$$

nov 4-10:31

$$\cos\theta = \frac{\vec{OR} \cdot \vec{OQ}}{|\vec{OR}||\vec{OQ}|} = \frac{1}{\sqrt{3}\cdot\sqrt{3}} = \frac{1}{3}$$



$\theta \approx 70{,}5^\circ$

nov 4-10:32

Vinkelräta:

$$\bar{u} \perp \bar{v}$$

Parallella:

$$\bar{u} \parallel \bar{v}$$

nov 4-10:38

Olika namn:

vinkelrät =

ortogonal =

normal

nov 4-10:40

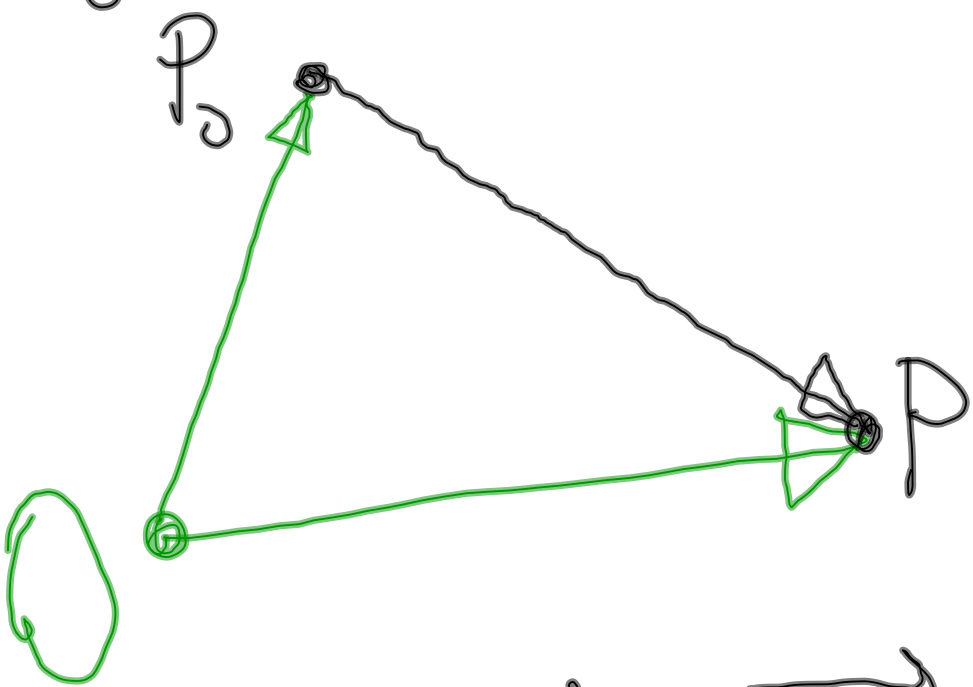
Plan

P_0 given punkt i planet (x_0, y_0, z_0)

P en rörlig punkt (x, y, z)

nov 4-10:42

$$\overrightarrow{P_0P} = \overrightarrow{OP} - \overrightarrow{OP_0}$$



$$\overrightarrow{OP_0} + \overrightarrow{P_0P} = \overrightarrow{OP}$$

nov 4-10:42

$$\overrightarrow{P_0P} \circ \bar{n} = 0$$

$$\left(\Longleftrightarrow \overrightarrow{P_0P} \perp \bar{n} \right)$$

$$\Longleftrightarrow \left(\overrightarrow{OP} - \overrightarrow{OP_0} \right) \circ \bar{n} = 0$$

$$\Longleftrightarrow \overrightarrow{OP} \circ \bar{n} - \overrightarrow{OP_0} \circ \bar{n} = 0$$

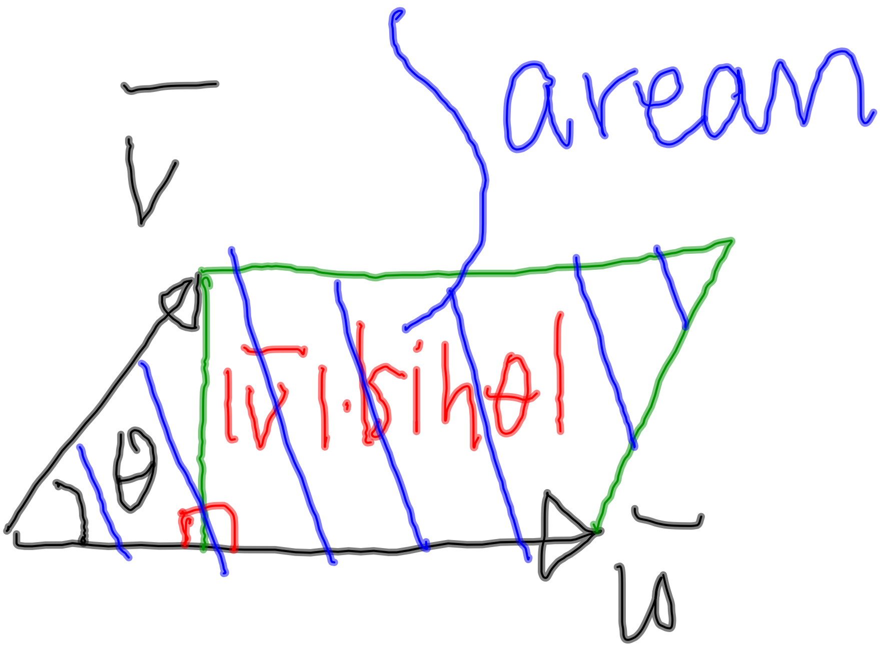
$$\Longleftrightarrow \overrightarrow{OP} \circ \bar{n} = \overrightarrow{OP_0} \circ \bar{n}$$

nov 4-10:44

Vektorprodukt

Tolka längden:

$$|\bar{u}| \cdot |\bar{v}| \cdot |\sin\theta|$$



nov 4-11:03

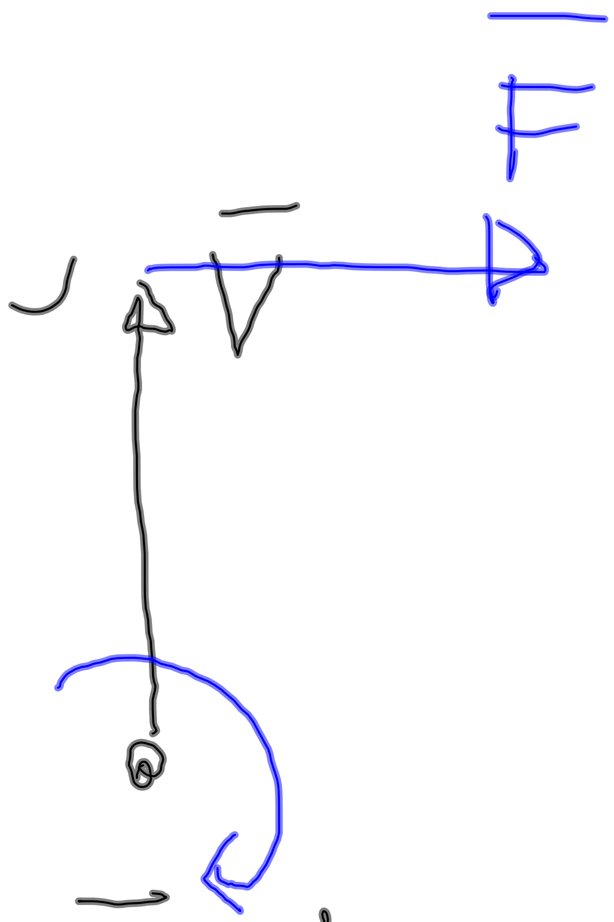
$$\bar{u} \times \bar{v} = 0 \iff$$

$$\bar{u} \parallel \bar{v}$$

nov 4-11:06

Moment

$\bar{F}$



$\bar{F} \times \bar{v}$ är momentvektorn

nov 4-11:11

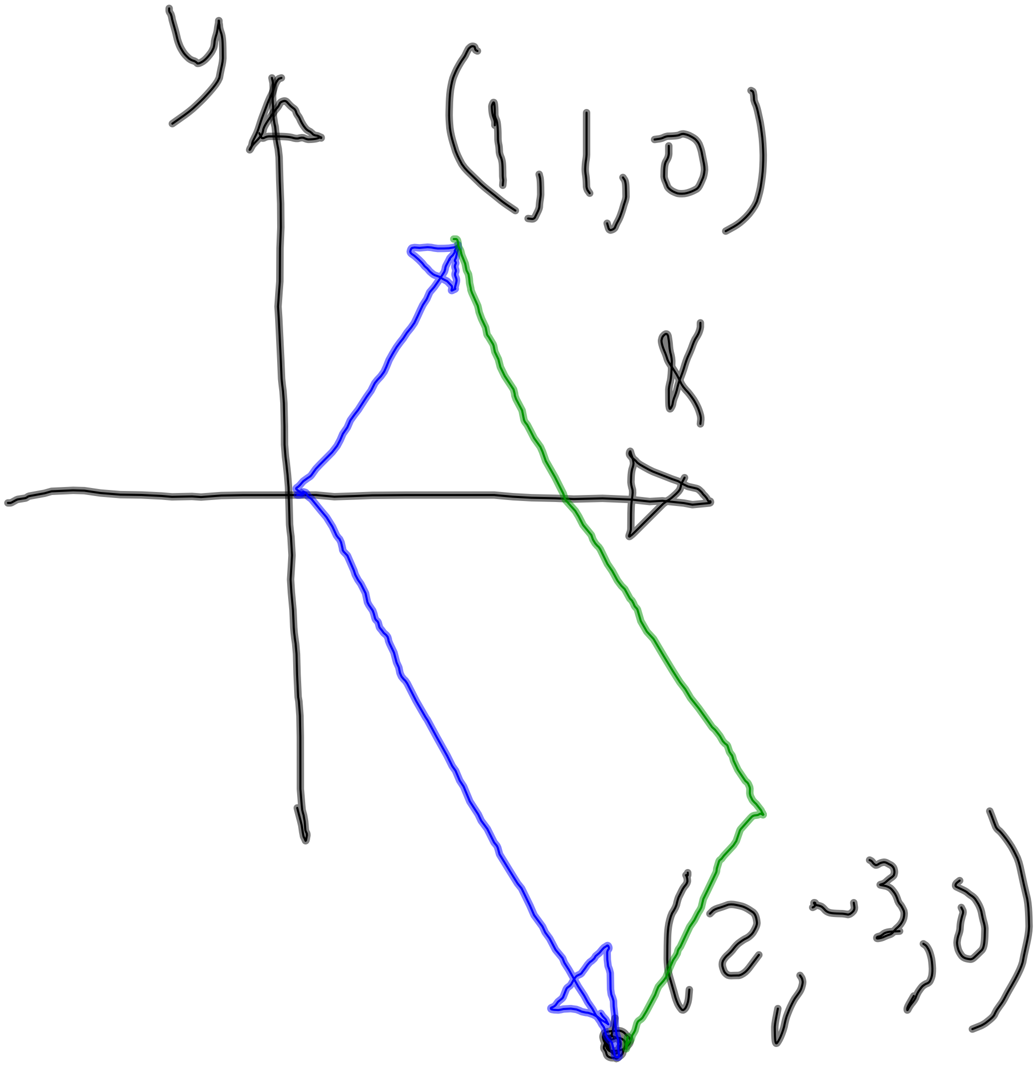
Korrektion

$$\bar{u} \times \bar{v} = (y_1 z_2 - z_1 y_2, \; z_1 x_2 - x_1 z_2, \; x_1 y_2 - y_1 x_2)$$

nov 4-11:17

Testa ett exempel:

$(1,1,0) \times (2,-3,0)$



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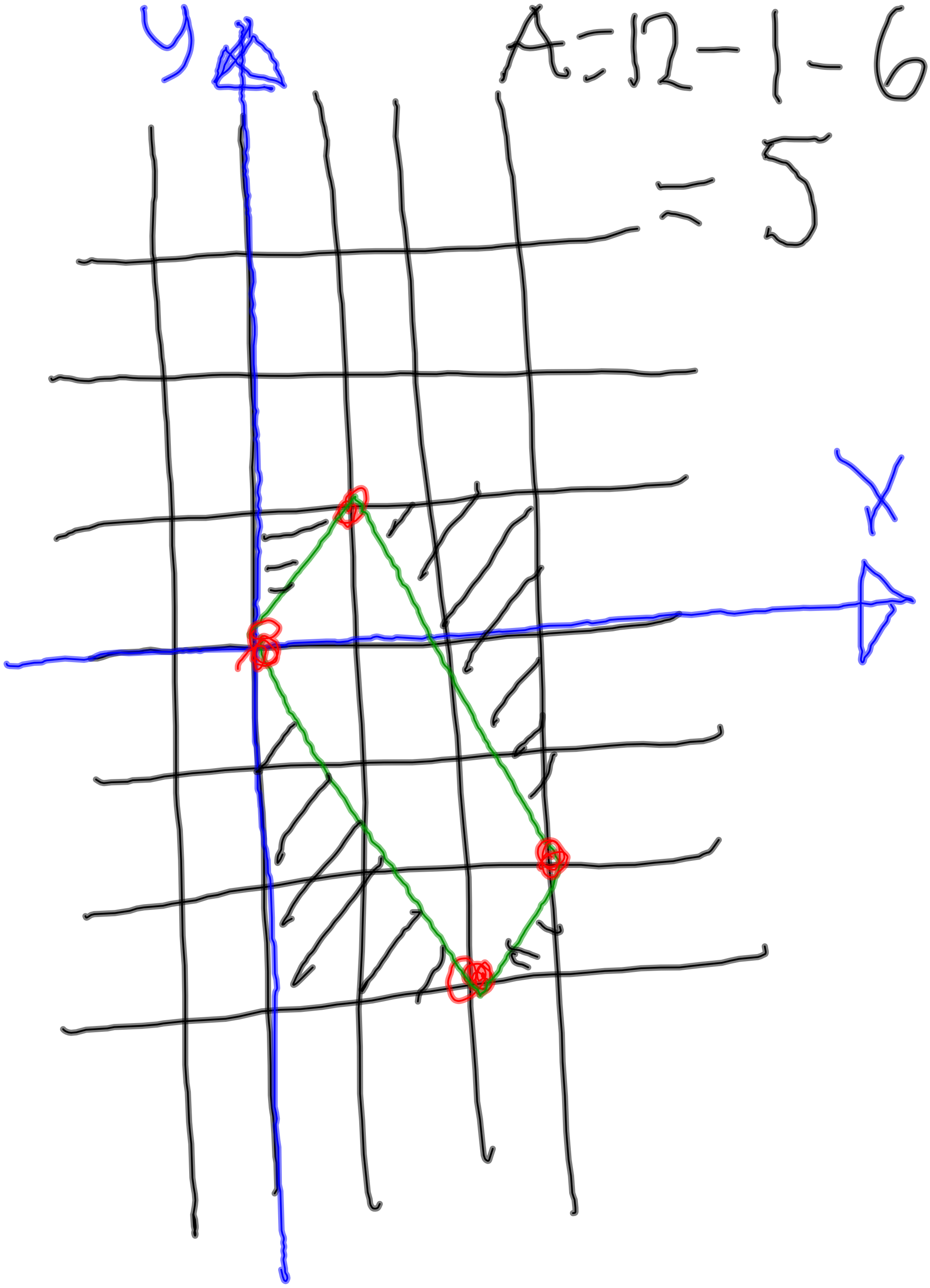
$$\begin{pmatrix} 1\cdot 0 - 0\cdot(-3), \\ 0\cdot 2 - 1\cdot 0, \\ 1\cdot(-3) - 1\cdot 2 \end{pmatrix}$$

$$= (0, 0, -5)$$

nov 4-11:20

Kolla arean

$A = 12 - 1 - 6$
$= 5$



nov 4-11:24

Ekvation för plan

$$ax+by+cz=d$$

där $\bar{n}=(a,b,c)$

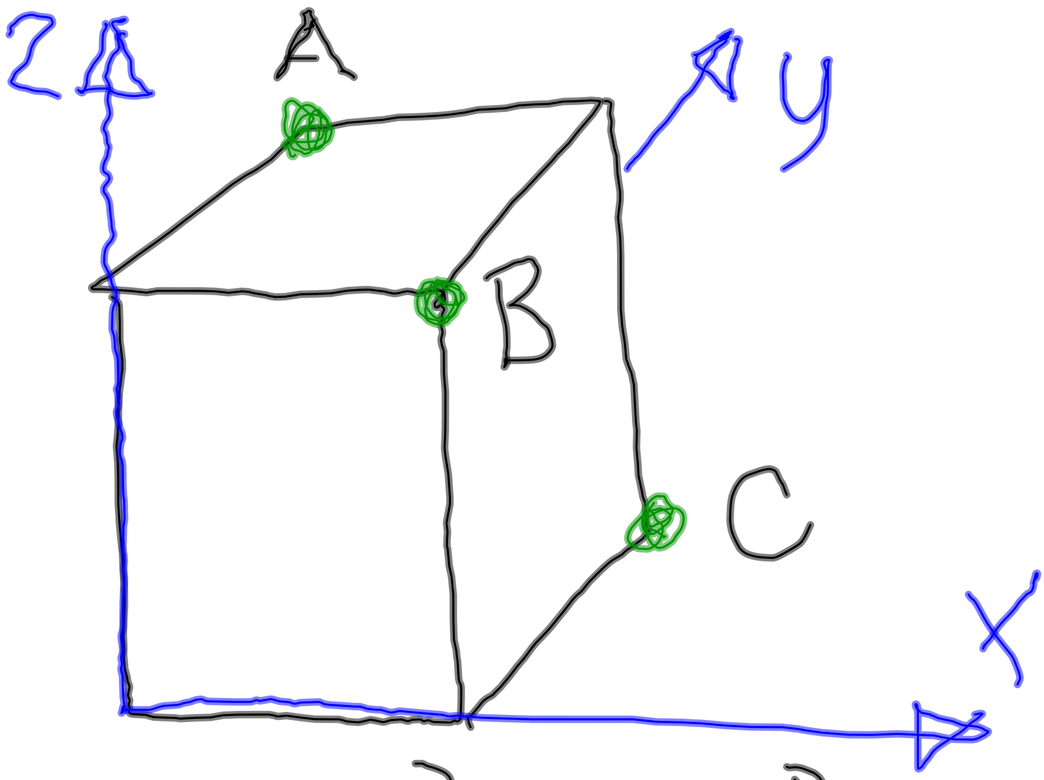
nov 4-11:28

Normalvektorn $\bar{n}$ kan fås som

$$\bar{n} = \bar{u} \times \bar{v}$$

där $\bar{u}$ och $\bar{v}$ är vektorer i planet.

nov 4-11:30



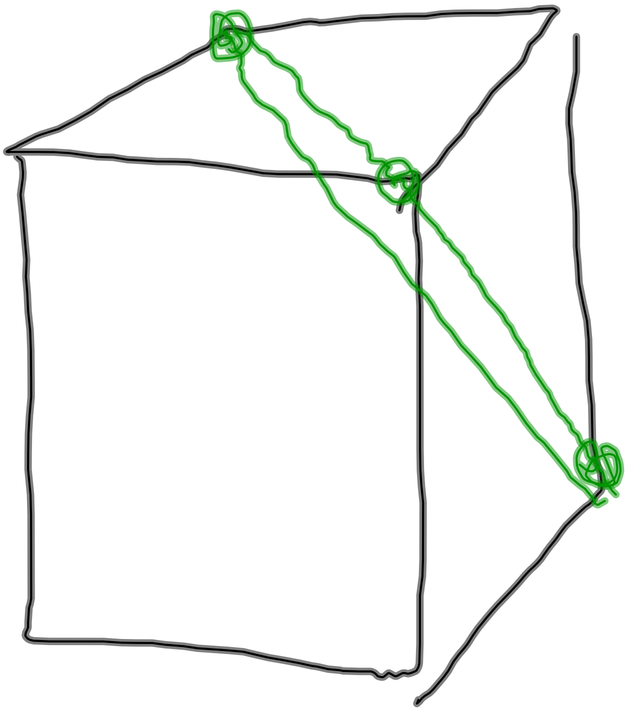
$$\bar{n} = \overrightarrow{AC} \times \overrightarrow{AB}$$

$$A = (0, 1, 1)$$

$$B = (1, 0, 1)$$

$$C = (1, 1, 0)$$

nov 4-11:31

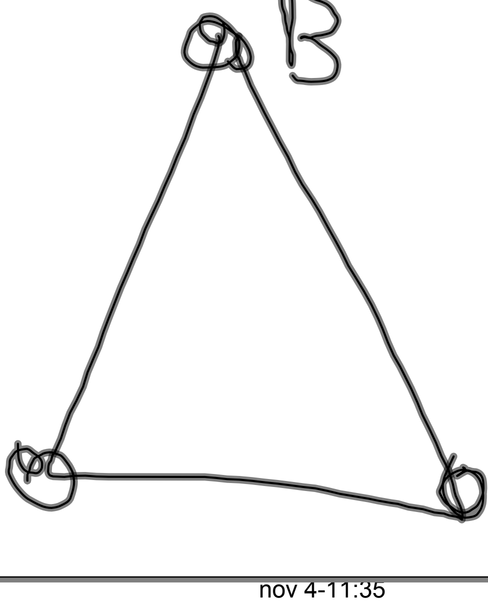


$\sqrt{2}$

$||$

$|AB| = |BC| = |AC|$

$\Rightarrow \theta = 60^\circ$



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$$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| =$$

$$\left|\overrightarrow{AB}\right| \cdot \left|\overrightarrow{AC}\right| \cdot \left|\sin 60^\circ\right|$$

$$= \sqrt{2} \cdot \sqrt{2} \cdot \frac{\sqrt{3}}{2} =$$

$$= \sqrt{3}$$

nov 4-11:37

Vi kan sätta in A, B och C i ekvationen

$$ax+by+cz=d$$

och får tre ekvationer i a, b, c, d.

nov 4-11:38

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= (1,0,1) - (0,1,1)$$

$$= (1,-1,0)$$

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$= (1,1,0) - (0,1,1)$$

$$= (1,0,-1)$$

nov 4-11:40

$$\overrightarrow{AB} \times \overrightarrow{AC} =$$

$$(1,-1,0) \times (1,0,-1)$$

$$= \Big((-1)\cdot(-1) - 0\cdot 0,$$

$$, 0\cdot 1 - 1\cdot(-1),$$

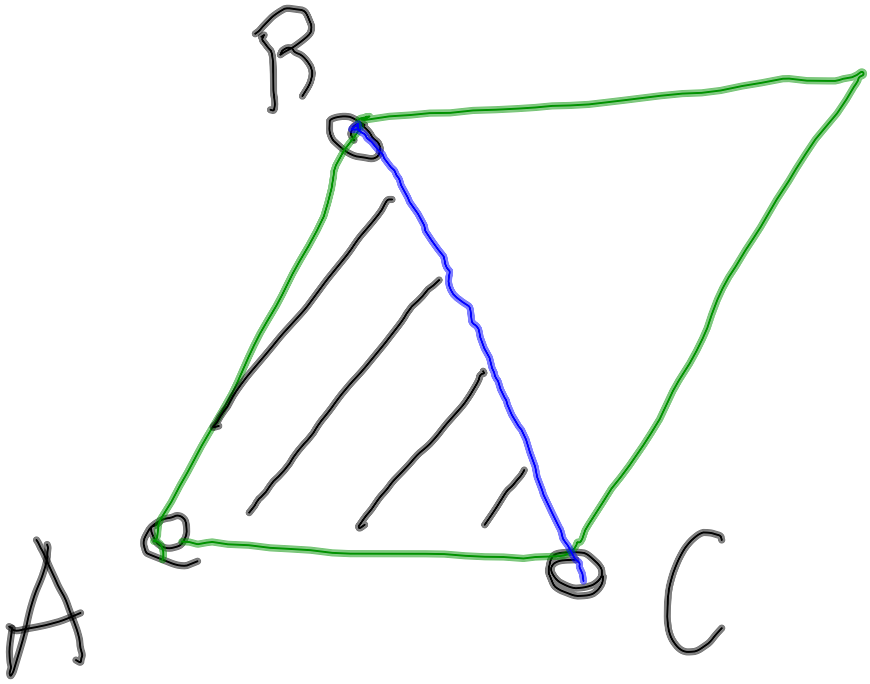
$$, 1\cdot 0 - (-1)\cdot 1\Big)$$

nov 4-11:41

$$= (1, 1, 1)$$

$$|(1,1,1)| = \sqrt{(1,1,1)\cdot(1,1,1)} = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

nov 4-11:43



Arean av ΔABC

är $\frac{1}{2}\sqrt{3}$

nov 4-11:44